Bologna, 2 October '06

Neutrino
Masses & Mixings

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In honour of Barry Barish and Sheldon Glashow

In the last decade data on ν oscillations have added some (badly needed) fresh experimental input in particle physics

ν masses are not all vanishing but they are very small

ν mixing angles follow a different pattern from quark mixings

For ν masses and mixings we do not have so far a Standard Model: many possibilities are still open.

In fact this is the case also for quark and charged leptons; we do not have a theory of flavour that explains the observed spectrum, mixings and CP violation.

ν's are interesting because they can provide new clues on this important problem

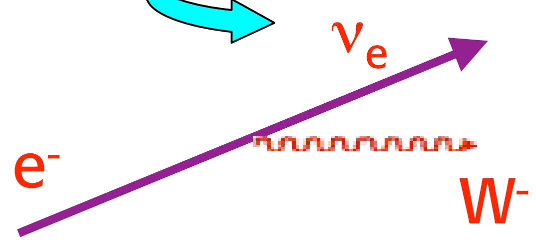
ν Oscillations Imply Different ν Masses

ν_e: same weak isospin doublet as e^-

flavour mass

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

U: mixing matrix



$U = U_{P\text{-}MNS}$

Pontecorvo
Maki, Nakagawa, Sakata

$$\nu_e = \cos\theta\ \nu_1 + \sin\theta\ \nu_2$$
$$\nu_\mu = -\sin\theta\ \nu_1 + \cos\theta\ \nu_2$$

e.g 2 flav.

Stationary source:

Stodolsky

$\nu_{1,2}$: different mass, different x-dep:

$$\nu_a(x) = e^{ip_a x}\ \nu_a \qquad p_a^2 = E^2 - m_a^2$$

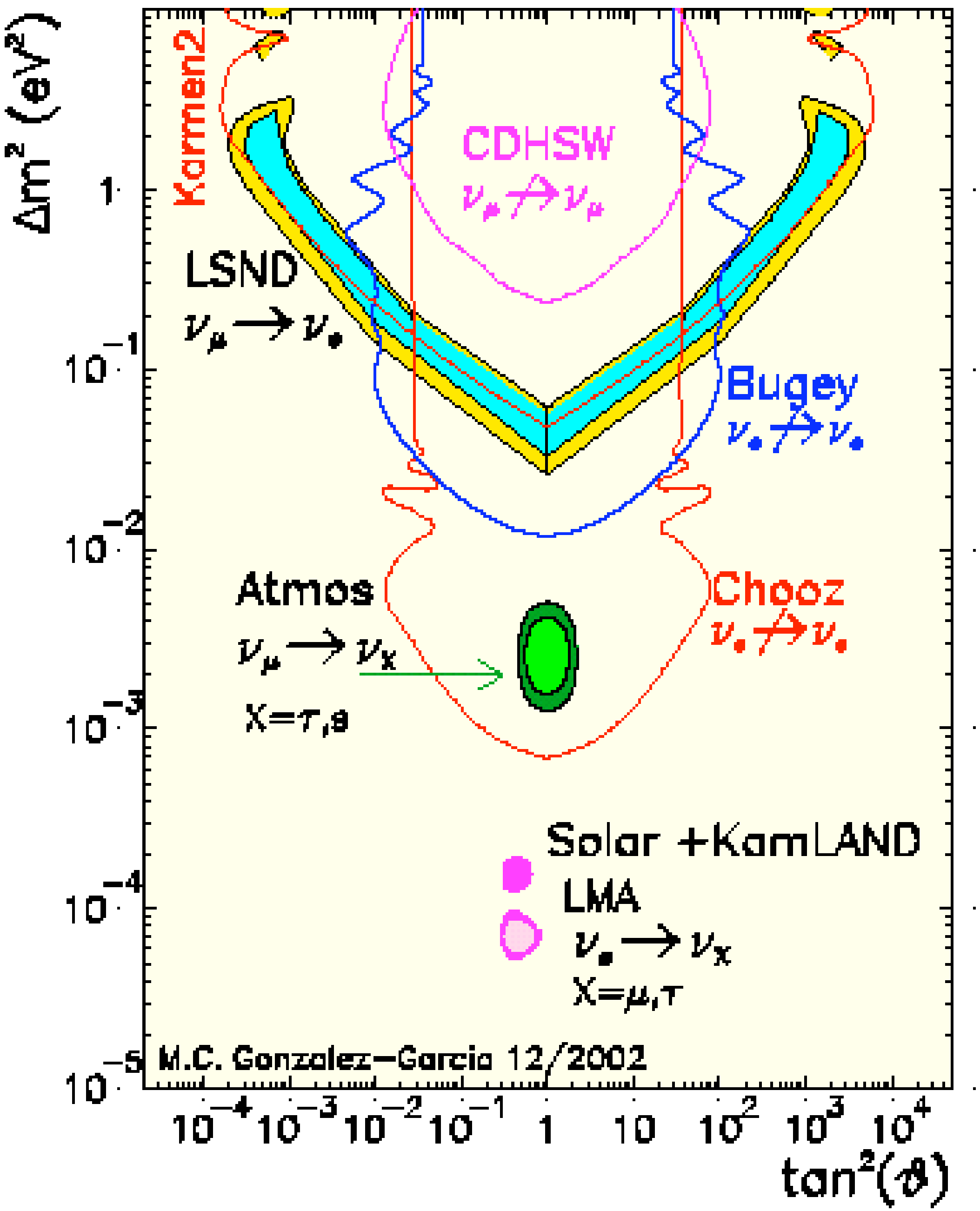
$$P(\nu_e <-> \nu_\mu) = |< \nu_\mu(L)|\ \nu_e>|^2 = \sin^2(2\theta)\cdot\sin^2(\Delta m^2 L/4E)$$

At a distance L, ν_μ from μ^- decay can produce e^- via charged weak interact's

Solid evidence for solar and atmosph. ν oscillations (+LSND unclear)

Δm^2 values fixed:
$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$,
$\Delta m^2_{sol} \sim 8\ 10^{-5}\ eV^2$
($\Delta m^2_{LSND} \sim 1\ eV^2$)

mixing angles:
θ_{12} (solar) large
θ_{23} (atm) large,~maximal
θ_{13} (CHOOZ) small



The current experimental situation is still unclear

- LSND: true or false? -> MiniBooNE soon will tell
- what is the absolute scale of ν masses?
- no detection of $0\nu\beta\beta$ (proof that ν's are Majorana) ••••••

Different classes of models are still possible:

If LSND true

sterile ν(s)??

CPT violat'n??

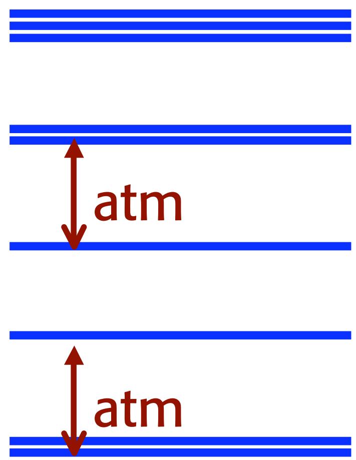
- "3-1" or "3-n"

$\nu_{sterile}$

If LSND false → 3 light ν's are OK

We assume this case here

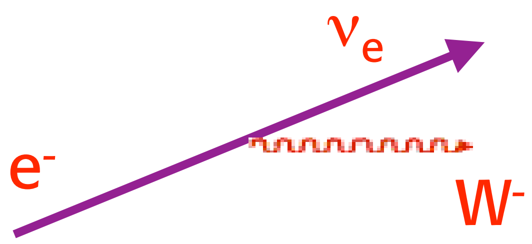
- Degenerate ($m^2 \gg \Delta m^2$)
- Inverse hierarchy
- Normal hierarchy



3-ν Models

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = U^{+}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

flavour mass



$U = U_{P\text{-}MNS}$

Pontecorvo
Maki, Nakagawa, Sakata

In basis where e^-, μ^-, τ^- are diagonal:

δ: CP violation

$$U = \begin{bmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{bmatrix}\begin{bmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{bmatrix}\begin{bmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{bmatrix} \sim$$

s = solar: large

CHOOZ: $|s_{13}| <\sim 0.2$

$$\sim \begin{bmatrix}c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta}\\ \ldots & \ldots & c_{13}s_{23}\\ & & c_{13}c_{23}\end{bmatrix}$$

atm.: ~ max

$$U = \begin{bmatrix}\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0\\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}}\\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\end{bmatrix}$$

(some signs are conventional)

In general: $U = U^{+}_{e}U_{\nu}$

$$m_\nu \sim U^* \begin{pmatrix} e^{i\alpha_1} m_1 & 0 & 0 \\ 0 & e^{i\alpha_2} m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^+$$

$L^T m_\nu L$

In general 9 parameters:
3 masses, 3 angles,
3 phases

For $s_{13} \sim 0$:

$0\nu\beta\beta$

$$m_\nu \sim \begin{pmatrix} m_1 c^2 + m_2 s^2 & (m_1 - m_2) cs/\sqrt{2} & (m_1 - m_2) cs/\sqrt{2} \\ \dots & (m_1 s^2 + m_2 c^2 + m_3)/2 & (m_1 s^2 + m_2 c^2 - m_3)/2 \\ \dots & \dots & (m_1 s^2 + m_2 c^2 + m_3)/2 \end{pmatrix}$$

Note:
- m_ν is symmetric
- phases included in m_i

Relation between masses and frequencies:

$P(\nu_e \leftrightarrow \nu_\mu) = P(\nu_e \leftrightarrow \nu_\tau) = 1/2 \sin^2 2\theta_{12} \cdot \sin^2 \Delta_{sun}$

$P(\nu_\mu \leftrightarrow \nu_\tau) = \sin^2 \Delta_{atm} - 1/4 \sin^2 2\theta_{12} \cdot \sin^2 \Delta_{sun}$

$$\Delta_{sun} = \frac{m_2^2 - m_1^2}{4E} L \quad ; \quad \Delta_{atm} = \frac{m_3^2 - m_{1,2}^2}{4E} L$$

In our def.: $\Delta_{sun} > 0$, $\Delta_{atm} >$ or < 0

Defining:

$$\Delta m^2_{atm} = m_3^2 - m_2^2 > \text{or} < 0$$

$$\Delta m^2_{sol} = m_2^2 - m_1^2 > 0$$

one has:

$$m_3^2 = \overline{m^2} + \frac{2}{3}\Delta m^2_{atm} + \frac{1}{3}\Delta m^2_{sol}$$

$$m_2^2 = \overline{m^2} - \frac{1}{3}\Delta m^2_{atm} + \frac{1}{3}\Delta m^2_{sol}$$

$$m_1^2 = \overline{m^2} - \frac{1}{3}\Delta m^2_{atm} - \frac{2}{3}\Delta m^2_{sol}$$

and

$\overline{m^2} >> \left|\Delta m^2_{atm}\right| > \Delta m^2_{sol}$ degenerate

$\Delta m^2_{atm} < 0$ inverse hierarchy

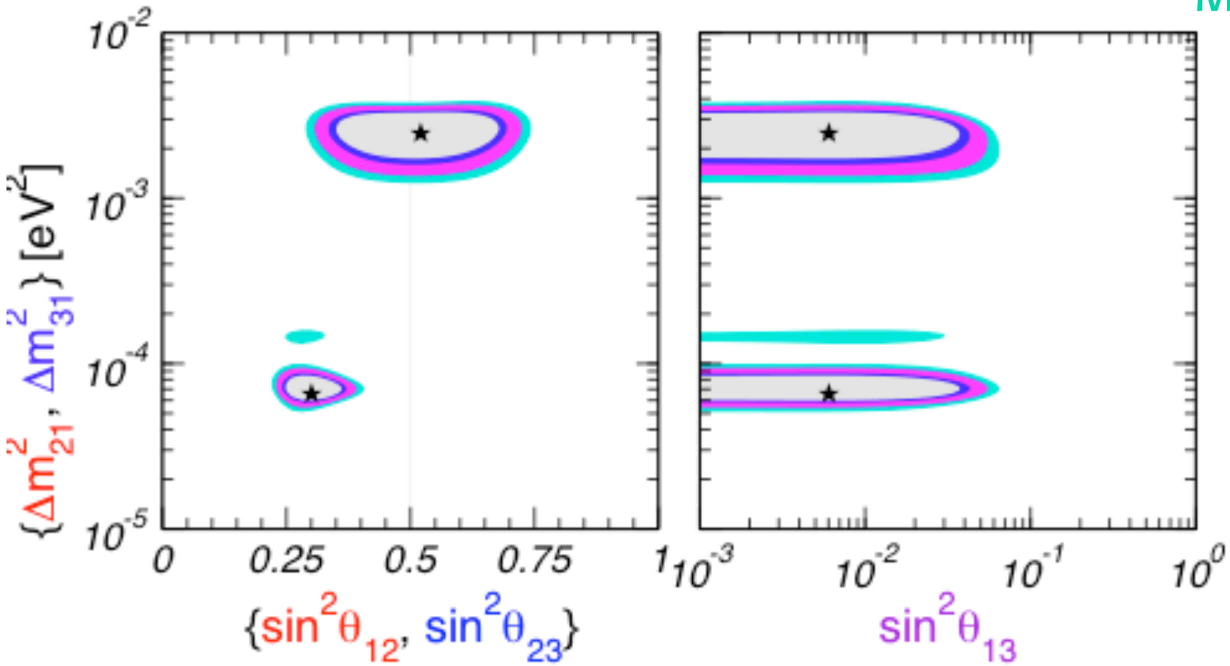
$\Delta m^2_{atm} > 0$ normal hierarchy

Neutrino oscillation parameters

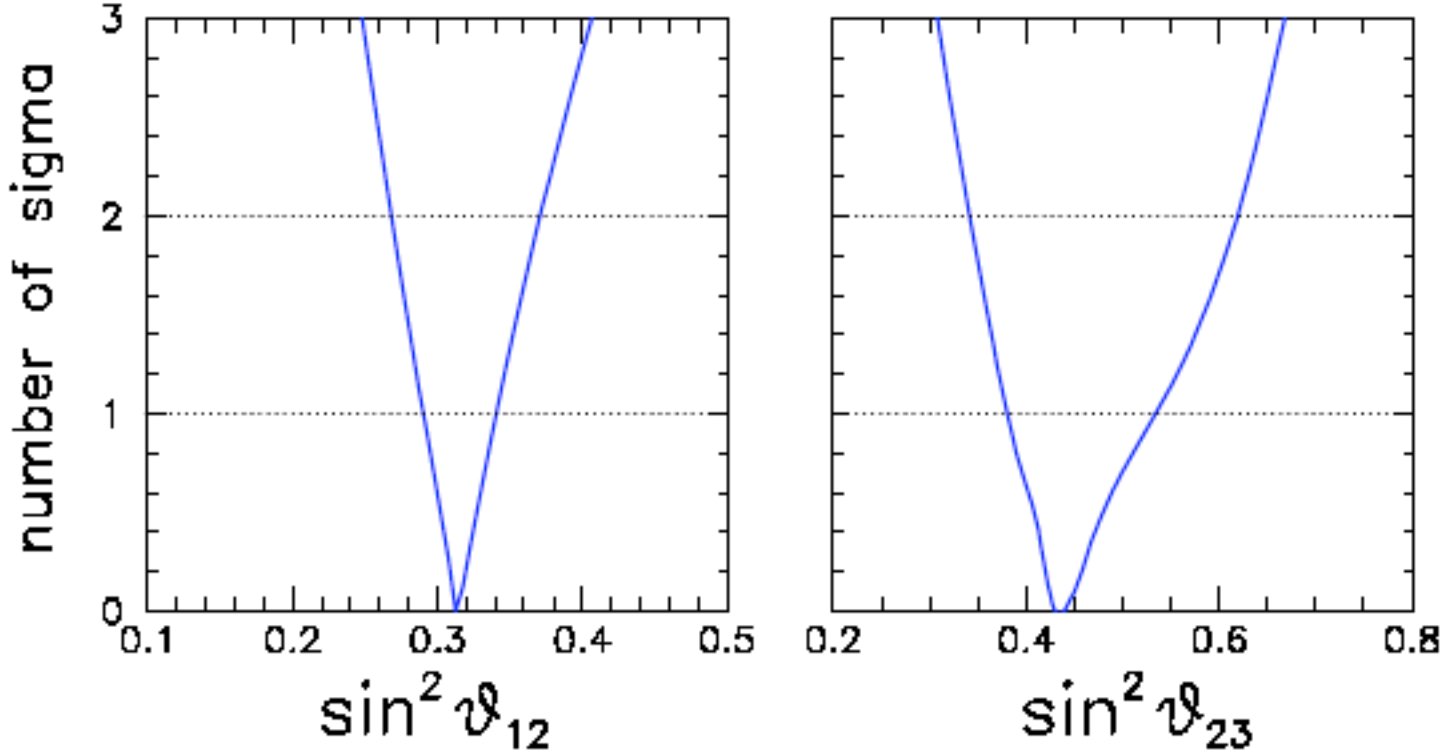
- 2 distinct frequencies
- 2 large angles, 1 small

parameter	best fit	2σ	3σ	5σ
$\Delta m^2_{21}\ [10^{-5}\text{eV}^2]$	6.9	6.0–8.4	5.4–9.5	2.1–28
$\Delta m^2_{31}\ [10^{-3}\text{eV}^2]$	2.6	1.8–3.3	1.4–3.7	0.77–4.8
$\sin^2\theta_{12}$	0.30	0.25–0.36	0.23–0.39	0.17–0.48
$\sin^2\theta_{23}$	0.52	0.36–0.67	0.31–0.72	0.22–0.81
$\sin^2\theta_{13}$	0.006	$\lesssim 0.035$	$\lesssim 0.054$	$\lesssim 0.11$

Maltoni et al '04



Fogli et al '05



2σ ranges 95%

very precise (KamLAND)
very close to 1/3

$$\delta m^2 = 7.92\,(1^{+0.09}_{-0.09}) \times 10^{-5}\ \mathrm{eV}^2$$

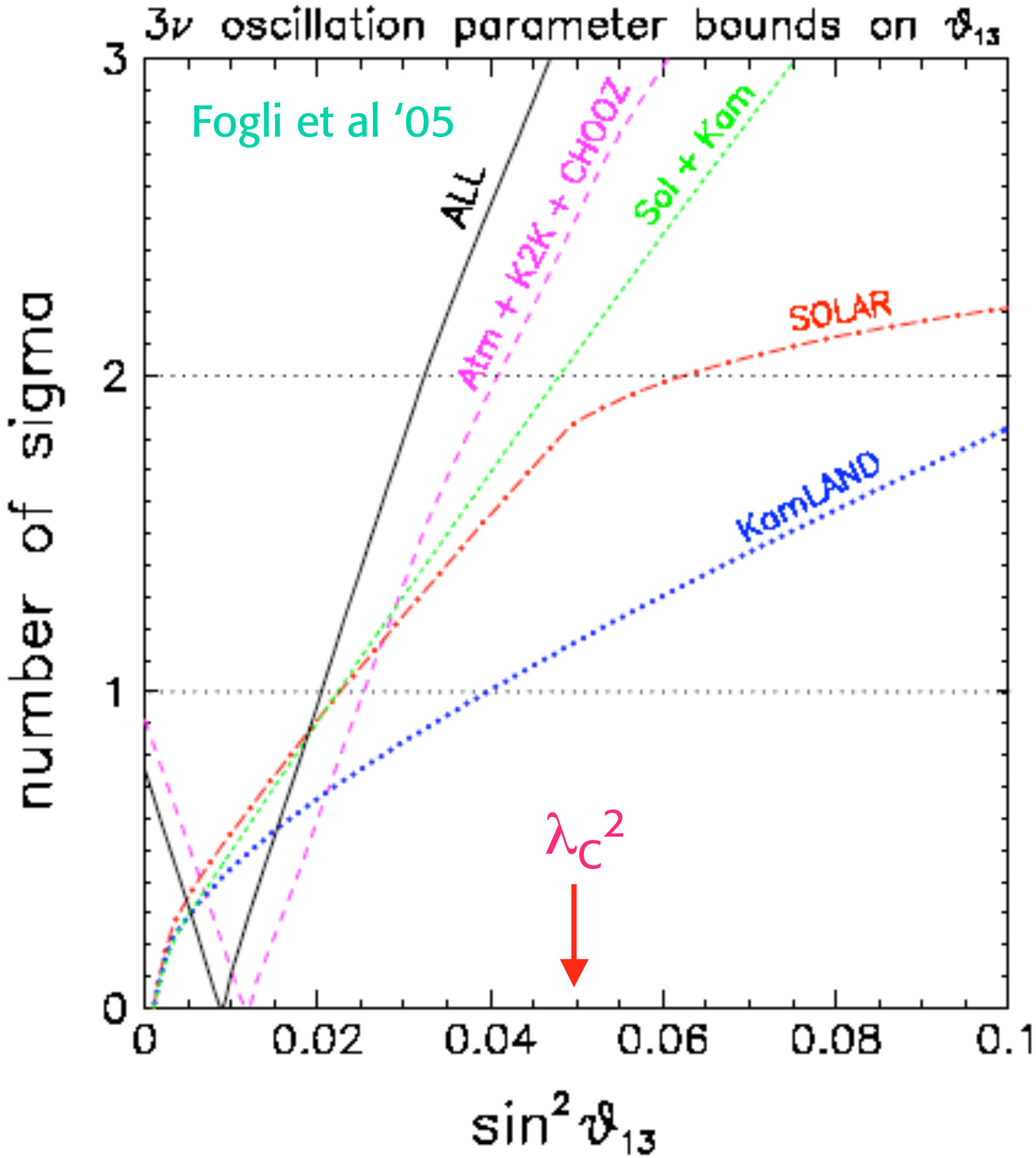
$$\Delta m^2 = 2.4\,(1^{+0.21}_{-0.26}) \times 10^{-3}\ \mathrm{eV}^2$$

$$\sin^2\theta_{12} = 0.314\,(1^{+0.18}_{-0.15})$$

$$\sin^2\theta_{23} = 0.44\,(1^{+0.41}_{-0.22})$$

$$\sin^2\theta_{13} < 3.2 \times 10^{-2}$$

θ_{13} bounds



ν oscillations measure Δm^2. What is m^2?

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$; $\Delta m^2_{sun} \sim 8\ 10^{-5}\ eV^2$

- Direct limits

End-point tritium β decay (Mainz, Troitsk)

$m_{\text{"}\nu e\text{"}} < 2.2$ eV

$m_{\text{"}\nu\mu\text{"}} < 170$ KeV

$m_{\text{"}\nu\tau\text{"}} < 18.2$ MeV

$m_{ee} = |\Sigma\, U_{ei}^2\, m_i|$

- $0\nu\beta\beta$ $m_{ee} < 0.3 - 0.7 - ?$ eV (nucl. matrix elmnts)

Evidence of signal? Klapdor-Kleingrothaus

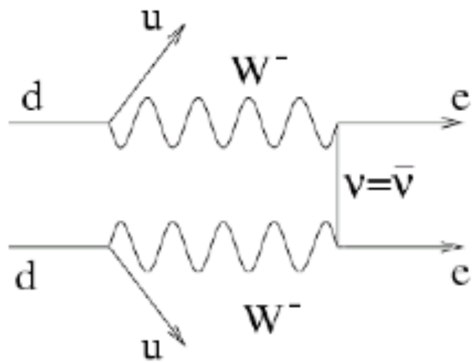
- Cosmology $\Omega_\nu\, h^2 \sim \Sigma_i m_i\, /94\text{eV}$ $(h^2 \sim 1/2)$

$\Sigma_i m_i < 0.17\text{-}0.68\text{-}2.1$ eV (dep. on data&priors)

→ Any ν mass $< 0.06\text{-}0.23\text{-}0.7$ eV

WMAP, 2dFGRS, Ly-α

$0\nu\beta\beta$ experiments



$$<m_\nu>^2 = \frac{1}{G(Q,Z)\,|M_{nucl}|^2\,\tau}$$

phase space

matrix elmnt
large uncrtnts

Pavan

Experiment	Isotope	$\tau_{1/2}^{0\nu}$ > [y]	range $<m_\nu>$ [eV]
Heidelberg Moscow 2001	^{76}Ge	1.9 10^{25}	0.3-2.5
IGEX 2002	^{76}Ge	1.57 10^{25}	0.3-2.5
Cuoricino 2005	^{130}Te	2 10^{24}	0.3-0.7
NEMO 2005	^{100}Mo	4.6 10^{23}	0.6-1.0

claimed evidence only by a part of the collaboration

started in 2003

$$m_{ee} = |\Sigma\; U_{ej}^2\; m_j\; e^{i\alpha j}|$$

Future: a factor ~ 10 improvement in next decade

ν oscillations measure Δm^2. What is m^2?

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- Direct limits

  $m_{"\nu e"} < 2.2$ eV ← End-point tritium β decay (Mainz, Troitsk)

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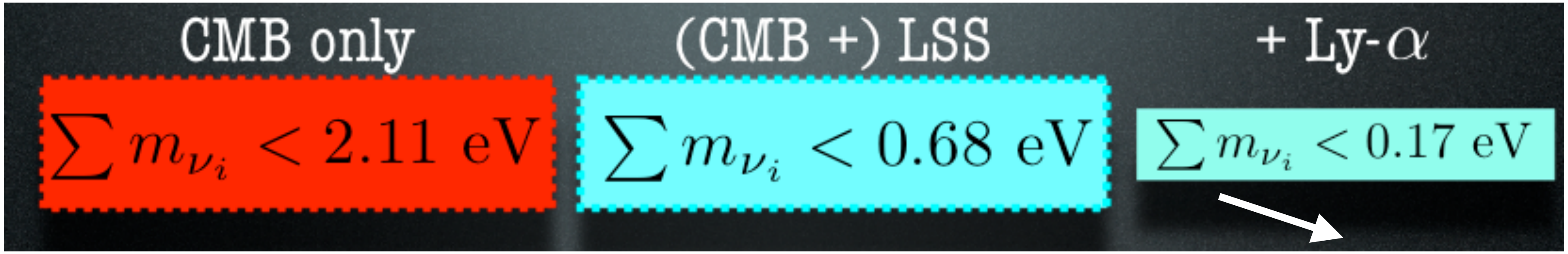
WMAP, SDSS, 2dFGRS, Ly-α

By itself CMB (eg WMAP) is only mildly sensitive to $\Sigma_i m_i$

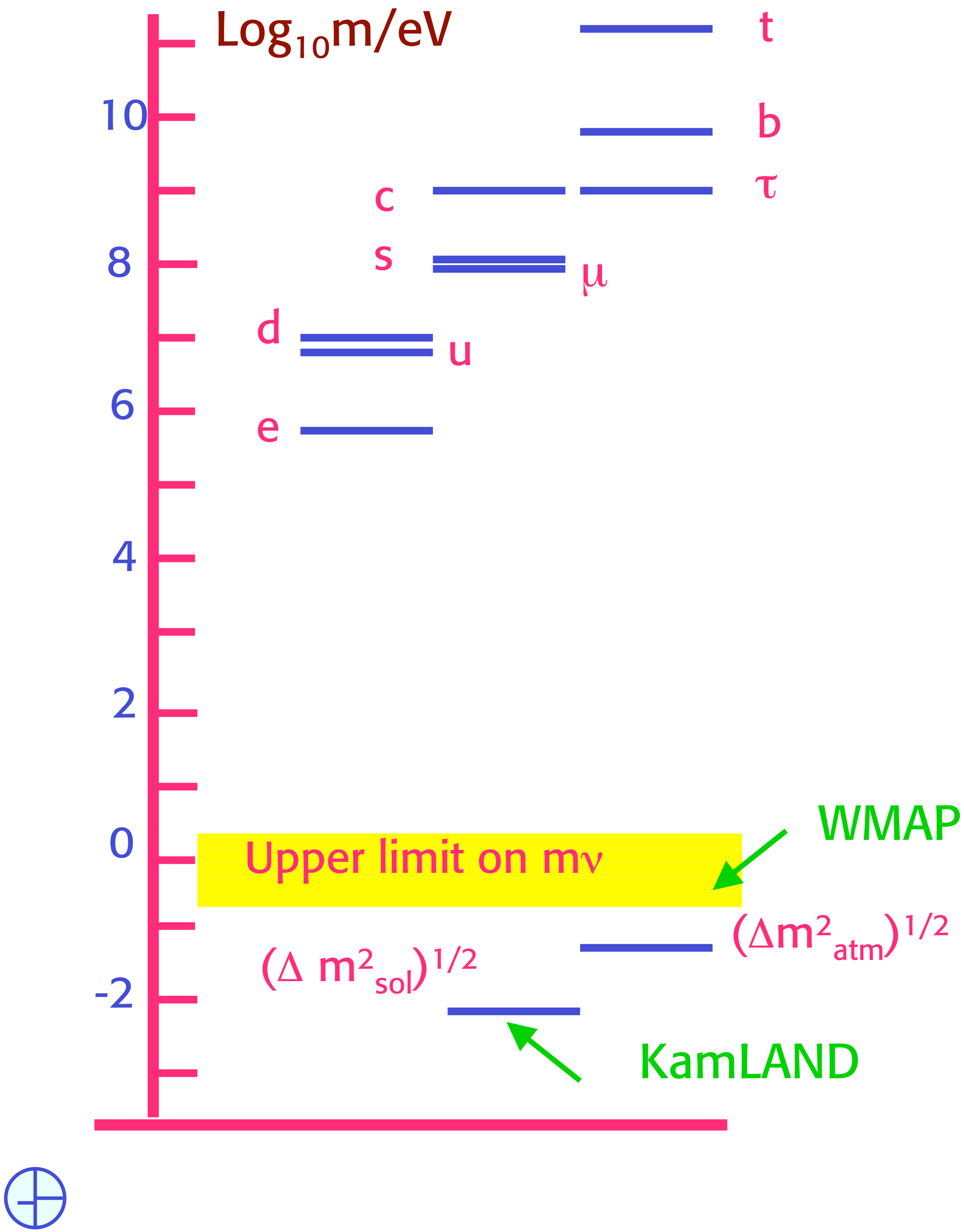
Only in combination with Large Scale Structure (2dFGRS, SDSS) the limit becomes stronger.

And even stronger by adding the Lyman alpha forest data (but some tension among the data).

95%cl



Seljac et al '06



Neutrino masses are really special!

$m_t/(\Delta m^2_{atm})^{1/2} \sim 10^{12}$

Massless ν's?

- no ν_R
- L conserved

Small ν masses?

- ν_R very heavy
- L not conserved

See-Saw Mechanism

Minkowski; Glashow;
Yanagida; Gell-Mann, Ramond , Slansky;
Mohapatra, Senjanovic…..

$M\nu^T{}_R\nu_R$ allowed by SU(2)xU(1)

Large Majorana mass M (as large as the cut-off)

$m_D\bar{\nu}_L\nu_R$

Dirac mass m from Higgs doublet(s)

$$\begin{array}{c|cc} & \nu_L & \nu_R \\ \hline \nu_L & 0 & m_D \\ \nu_R & m_D & M \end{array}$$

$M \gg m_D$

Eigenvalues

$$\nu_{light} = \frac{-m_D{}^2}{M} \,, \qquad \nu_{heavy} = M$$

sign conventional for fermions

In general ν mass terms are:

$$\mathcal{L}_\nu = \bar{L} h \nu_R H + \text{h.c.} + \nu_R^T M_R \nu_R + \nu_L^T \frac{\lambda}{M_L} \nu_L H H$$

Dirac
$m_D = hv$
$v = \langle 0|H|0\rangle$

Majorana

$$m = \frac{\lambda v^2}{M_L}$$

More general see-saw mechanism:

	ν_L	ν_R
ν_L	$\lambda v^2/M_L$	m_D
ν_R	m_D	M_R

$m_{light} \sim \dfrac{m_D^2}{M_R}$ and/or $\dfrac{\lambda v^2}{M_L}$

$m_{heavy} \sim M_R$

$m_{eff} = \nu^T_L m_{light} \nu_L$

A very natural and appealing explanation:

ν's are nearly massless because they are Majorana particles and get masses through L non conserving interactions suppressed by a large scale $M \sim M_{GUT}$

$$m_\nu \sim \frac{m^2}{M}$$

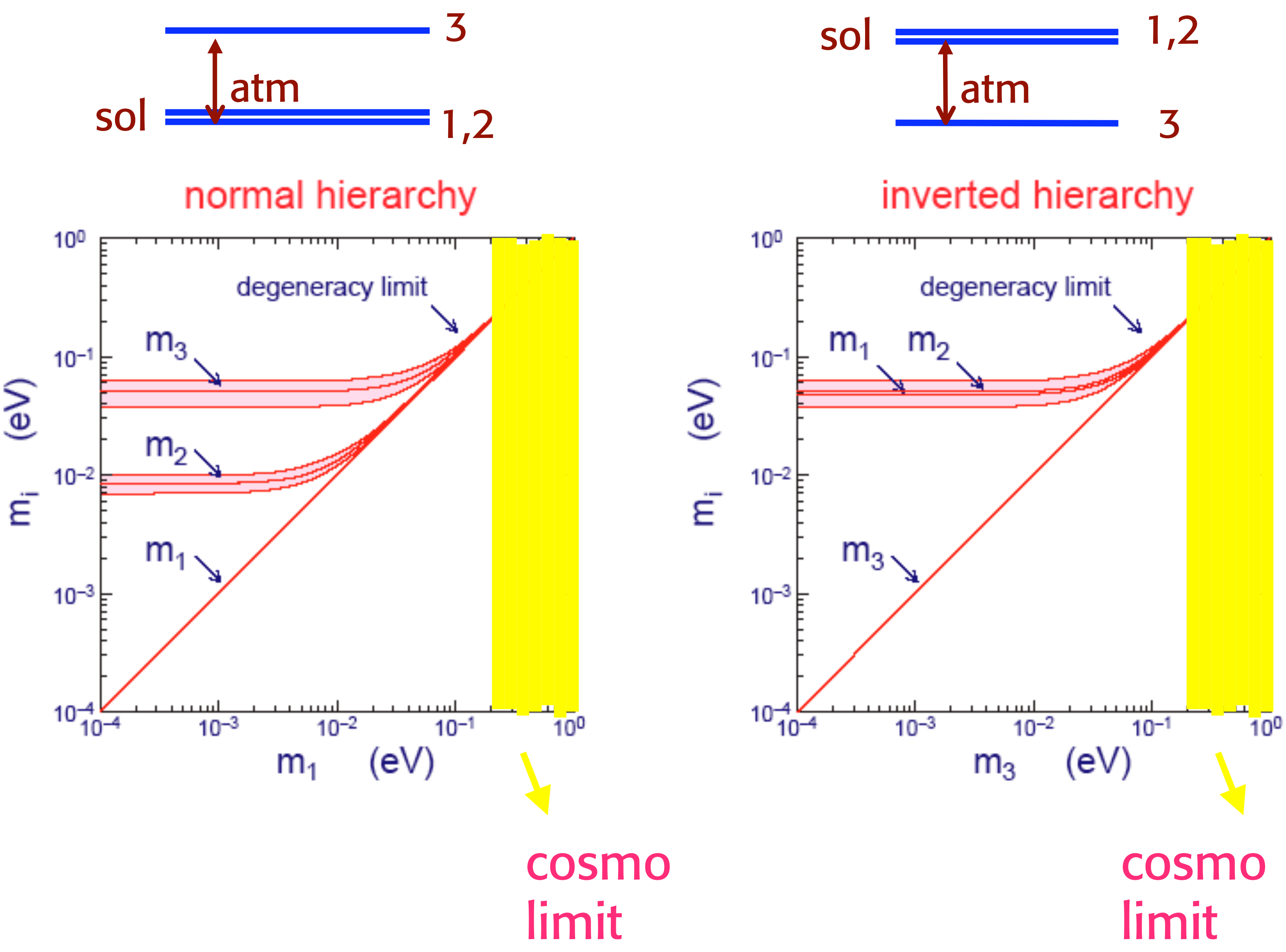
$m: \leq m_t \sim v \sim 200$ GeV
M: scale of L non cons.

Note:

$m_\nu \sim (\Delta m^2_{atm})^{1/2} \sim 0.05$ eV
$m \sim v \sim 200$ GeV

$M \sim 10^{14} - 10^{15}$ GeV

Neutrino masses are a probe of physics at M_{GUT} !



Only moderate degeneracy allowed

$0\nu\beta\beta$ would prove that L is not conserved and ν's are Majorana
Also can tell degenerate, inverted or normal hierarchy

$$|m_{ee}|=c_{13}^2\,[m_1 c_{12}^2+e^{i\alpha}m_2 s_{12}^2]+m_3 e^{i\beta}s_{13}^2$$

Degenerate: $\sim|m||c_{12}^2+e^{i\alpha}s_{12}^2|\sim|m|(0.3\text{-}1)$

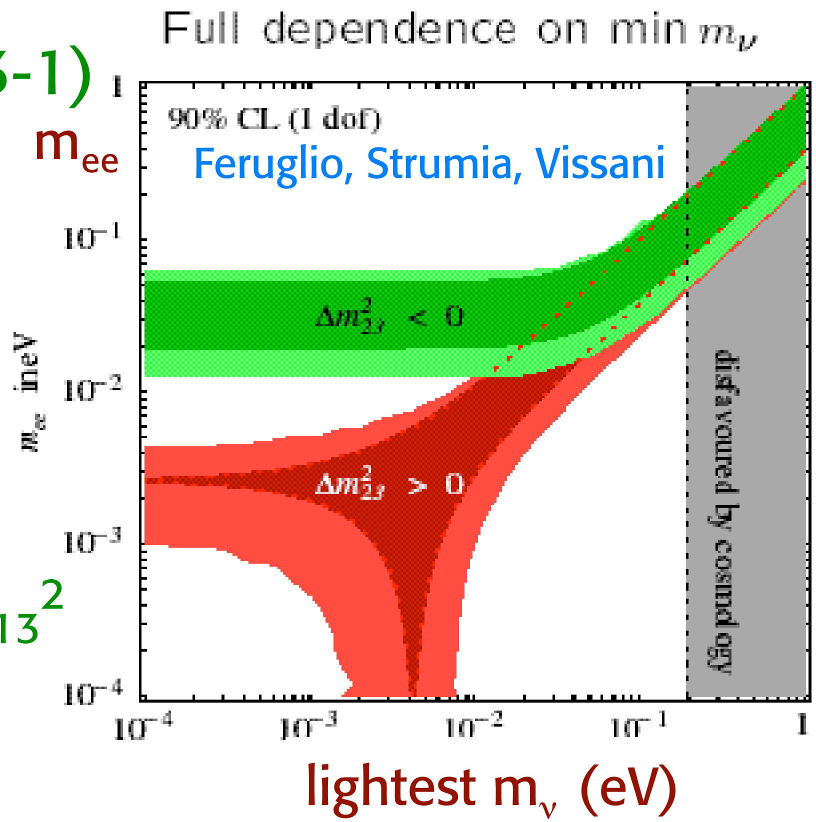
$|m_{ee}|\sim|m|\ (0.3\ \text{-}1)\leq< 0.23\text{-}1\ \text{eV}$

IH: $\sim(\Delta m^2_{atm})^{1/2}|c_{12}^2+e^{i\alpha}s_{12}^2|$

$|m_{ee}|\sim(1.6\text{-}5)\ 10^{-2}\ \text{eV}$

NH: $\sim(\Delta m^2_{sol})^{1/2}s_{12}^2+(\Delta m^2_{atm})^{1/2}e^{i\beta}s_{13}^2$

$|m_{ee}|\sim(\text{few})\ 10^{-3}\ \text{eV}$



Present exp. limit: $m_{ee}< 0.3\text{-}0.5$ eV
(and a hint of signal????? Klapdor Kleingrothaus)

Baryogenesis: a most attractive possibility

BG via Leptogenesis near the GUT scale

$T \sim 10^{12\pm3}$ GeV (after inflation)

Buchmuller,Yanagida, Plumacher, Ellis, Lola, Giudice et al, Fujii et al …..

Only survives if Δ(B-L)≠is not zero
(otherwise is washed out at T_{ew} by instantons)

Main candidate: decay of lightest ν_R ($M \sim 10^{12}$ GeV)

L non conserv. in ν_R out-of-equilibrium decay:
B-L excess survives at T_{ew} and gives the obs. B asymmetry.

Quantitative studies confirm that the range of m_i from ν oscill's is compatible with BG via (thermal) LG

In particular the bound was derived for hierarchy

$m_i < 10^{-1}$ eV

Buchmuller, Di Bari, Plumacher; Giudice et al; Pilaftsis et al; Hambye et al

Can be relaxed for degenerate neutrinos
So fully compatible with oscill'n data!!

Model building

Quality factors for models:

- Based on the most general lagrangian compatible with some simple symmetry or dynamical principle
- Should be complete: address at least charged leptons and neutrinos ($U_{P\text{-}NMS} = U^{+}_{e} U_{\nu}$, and the gauge symmetry connects ch. leptons and LH neutrinos)
- As many as possible small parameters (masses and mixings) should be naturally explained as a consequence.
- The necessary vev configuration should be a minimum of the most general potential for a region of parameter space
- The stability under radiative corrections and higher dim operators must be checked
- Simplicity, economy of fields and parameters, predictivity

Some recent work by our group

G.A., F. Feruglio, I. Masina, hep-ph/0402155,
G.A., F. Feruglio, hep-ph/0504165,hep-ph/0512103
G.A, R. Franceschini, hep-ph/0512202.

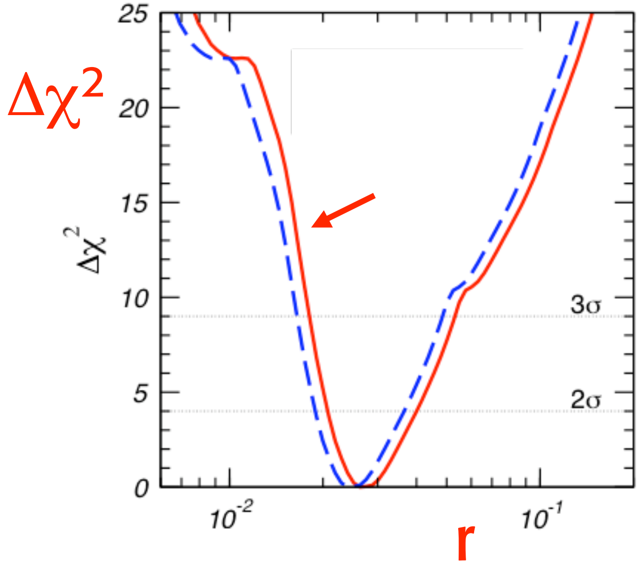
Reviews:

G.A., F. Feruglio, New J.Phys.6:106,2004 [hep-ph/0405048];
G.A., hep-ph/0410101; F. Feruglio, hep-ph/0410131

General remarks

- After KamLAND, SNO and WMAP.... not too much hierarchy is needed for ν masses:

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/30$



Precisely at 2σ: $0.025 < r < 0.049$

or

$m_{heaviest} < 0.2 - 0.7$ eV
$m_{next} > \sim 8\ 10^{-3}$ eV

For a hierarchical spectrum: $\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$

Comparable to: $\lambda_C \approx 0.22$ or $\sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$

Suggests the same "hierarchy" parameters for q, l, ν

⟶ e.g. θ_{13} not too small!

- Still large space for non maximal 23 mixing

3-σ interval $0.31 < \sin^2\theta_{23} < 0.72$

Maximal θ_{23} theoretically hard

- θ_{13} not necessarily too small

  probably accessible to exp.

Very small θ_{13} theoretically hard

Normal models: θ_{23} large but not maximal,
θ_{13} not too small (θ_{13} of order λ_C or λ_C^2)

Exceptional models: θ_{23} maximal and/or θ_{13} very small
or: a special value for θ_{12}....

A very exceptional model

Harrison, Perkins, Scott

A simple mixing matrix compatible with all present data

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

In the basis of diagonal ch. leptons:

$$m_\nu = U \mathrm{diag}(m_1, m_2, m_3) U^T$$

$$m_\nu = \frac{m_3}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{m_2}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{m_1}{6}\begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

Eigenvectors: $m_3 \to \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\quad m_2 \to \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad m_1 \to \frac{1}{\sqrt{6}}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

Note: mixing angles independent of mass eigenvalues

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Comparison with experiment:

At 1σ:

Fogli et al '05

$\sin^2\theta_{12} = 1/3$: 0.290-0.342

$\sin^2\theta_{23} = 1/2$: 0.39-0.53

$\sin^2\theta_{13} = 0$: < 0.02

The HPS mixing is clearly a very good approx. to the data!

Also called:
Tri-Bimaximal mixing

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

- For the HPS mixing matrix all mixing angles are fixed to particularly symmetric values

It is interesting to construct models that can naturally produce this highly ordered structure

Models based on the A4 discrete symmetry (even permutations of 1234) are very interesting Ma...;
GA, Feruglio hep-ph/0504165, hep-ph/0512103

Alternative models based on $SU(3)_F$ or $SO(3)_F$
Verzielas, G. Ross King

A4 is the discrete group of even perm's of 4 objects.
(the inv. group of a tetrahedron). It has 4!/2 = 12 elements.

An element is abcd which means 1234 --> abcd

C_1: 1 = 1234
C_2: T = 2314   ST = 4132   TS = 3241   STS = 1423
C_3: T^2 = 3124 ST^2 = 4213 T^2S = 2431   TST = 1342
C_4: S = 4321 T^2ST = 3412 TST^2 = 2143

Thus A4 transf.s can be written as:

1, T, S, ST, TS, T^2, TST, STS, ST^2, T^2S, T^2ST, TST^2

with: $S^2 = T^3 = (ST)^3 = 1$ [$(TS)^3 = 1$ also follows]

x, x′ in same class if

C_1, C_2, C_3, C_4 are equivalence classes   [$x' \sim gxg^{-1}$]   g: group element

A4 has only 4 irreducible inequivalent represt'ns: 1,1',1",3

Table of Multiplication:
1'x1'=1"; 1"x1"=1';1'x1"=1
3x3=1+1'+1"+3+3

A4 is well fit for 3 families!

Ch. leptons l ~ 3

e^c, μ^c, τ^c ~ 1, 1', 1"

In the (S-diag basis) consider 3: (a_1,a_2,a_3) $\xrightarrow{S}$ $(a_1,-a_2,-a_3)$

$\xrightarrow{T}$ (a_2,a_3,a_1)

For $3_1=(a_1,a_2,a_3)$, $3_2=(b_1,b_2,b_3)$ we have in $3_1 x 3_2$:

$$1 = a_1b_1 + a_2b_2 + a_3b_3$$

$$1' = a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3$$

$$1" = a_1b_1 + \omega a_2b_2 + \omega^2 a_3b_3$$

$$3 \sim (a_2b_3, a_3b_1, a_1b_2)$$

$$3 \sim (a_3b_2, a_1b_3, a_2b_1)$$

e.g. $1" = a_1b_1+\omega a_2b_2+\omega^2 a_3b_3 \xrightarrow{T} a_2b_2+\omega a_3b_3+\omega^2 a_1b_1 =$
$= \omega^2 [a_1b_1+\omega a_2b_2+\omega^2 a_3b_3]$

while, under S, 1" is inv.

Three singlet inequivalent represent'ns:

Recall:

$S^2 = T^3 = (ST)^3 = 1$

$$\begin{cases} 1: \; S=1,\, T=1 \\ 1': \; S=1,\, T=\omega \\ 1'': \; S=1,\, T=\omega^2 \end{cases}$$

$$\omega = \exp i\frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$
$$\omega^3 = 1$$
$$1 + \omega + \omega^2 = 0$$
$$\omega^2 = \omega^*$$

The only indep. 3-dim represent'n is obtained by:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(S-diag basis)

An equivalent form:

$$VV^\dagger = V^\dagger V = 1$$

$$S' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = VSV^\dagger \qquad T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{bmatrix} = VTV^\dagger \qquad V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

(T-diag basis)

Under A4

lepton doublets l ~ 3
e^c, μ^c, τ^c ~ 1, 1′, 1″ respectively
gauge singlet flavons ϕ, ϕ', ξ, (ξ') ~ 3, 3, 1,(1) respectively
driving fields (for SUSY version) ϕ_0, ϕ'_0, ξ_0 ~ 3, 3, 1

Additional symmetries: broken $U(1)_F$ symmetry (ch. lepton masses) with e^c, μ^c, τ^c charges (3 or 4,2,0)
and a discrete symmetry (dep. on versions) : for example
Z: (e^c, μ^c, τ^c)-> -i (e^c, μ^c, τ^c), l -> il, ϕ -> ϕ, (ξ,ϕ') -> - (ξ,ϕ')

The Yukawa interactions in the lepton sector are:

$$\mathcal{L}_Y = y_e e^c(\varphi l) + y_\mu \mu^c(\varphi l)'' + y_\tau \tau^c(\varphi l)' + x_a \xi(ll) + x_d(\varphi' ll) + h.c. + \ldots$$

⊕ Here is without see-saw (with see-saw is also OK: wait!)

Structure of the model

$$\mathcal{L}_Y = y_e e^c(\varphi l) + y_\mu \mu^c(\varphi l)'' + y_\tau \tau^c(\varphi l)' + x_a \xi(ll) + x_d(\varphi' ll) + h.c. + \ldots$$

shorthand: Higgs and cut-off scale Λ omitted, e.g.:

$$y_e e^c(\varphi l) \sim y_e e^c(\varphi l) h_d/\Lambda, \qquad x_a \xi(ll) \sim x_a \xi(l h_u l h_u)/\Lambda^2$$

$$\begin{aligned} \langle\varphi'\rangle &= (v', 0, 0) \\ \langle\varphi\rangle &= (v, v, v) \\ \langle\xi\rangle &= u \end{aligned}$$

$$m_l = v_d \frac{v}{\Lambda} \begin{pmatrix} y_e & y_e & y_e \\ y_\mu & y_\mu\omega^2 & y_\mu\omega \\ y_\tau & y_\tau\omega & y_\tau\omega^2 \end{pmatrix}$$

Spectrum free.
Diagonalized by U_e:

the big plus of A4

$$m_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix}$$

$$l \to \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} l = V l$$

⊕ From here it follows that U_{HPS} is the mixing matrix

m_ν in the basis of diagonal charged leptons is:

$$m_\nu\big|_{ldiag} \sim V^* \begin{bmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{bmatrix} V^* = \begin{pmatrix} a+2d/3 & -d/3 & -d/3 \\ -d/3 & 2d/3 & a-d/3 \\ -d/3 & a-d/3 & 2d/3 \end{pmatrix}$$

which in turn can be written as:

$$m_\nu\big|_{ldiag} \sim U^T \begin{bmatrix} a+d & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & -a+d \end{bmatrix} U$$

with:

$$U = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}$$

The crucial issue is to guarantee the strict alignment

$$\begin{aligned} \langle \varphi' \rangle &= (v', 0, 0) \\ \langle \varphi \rangle &= (v, v, v) \\ \langle \xi \rangle &= u \end{aligned}$$

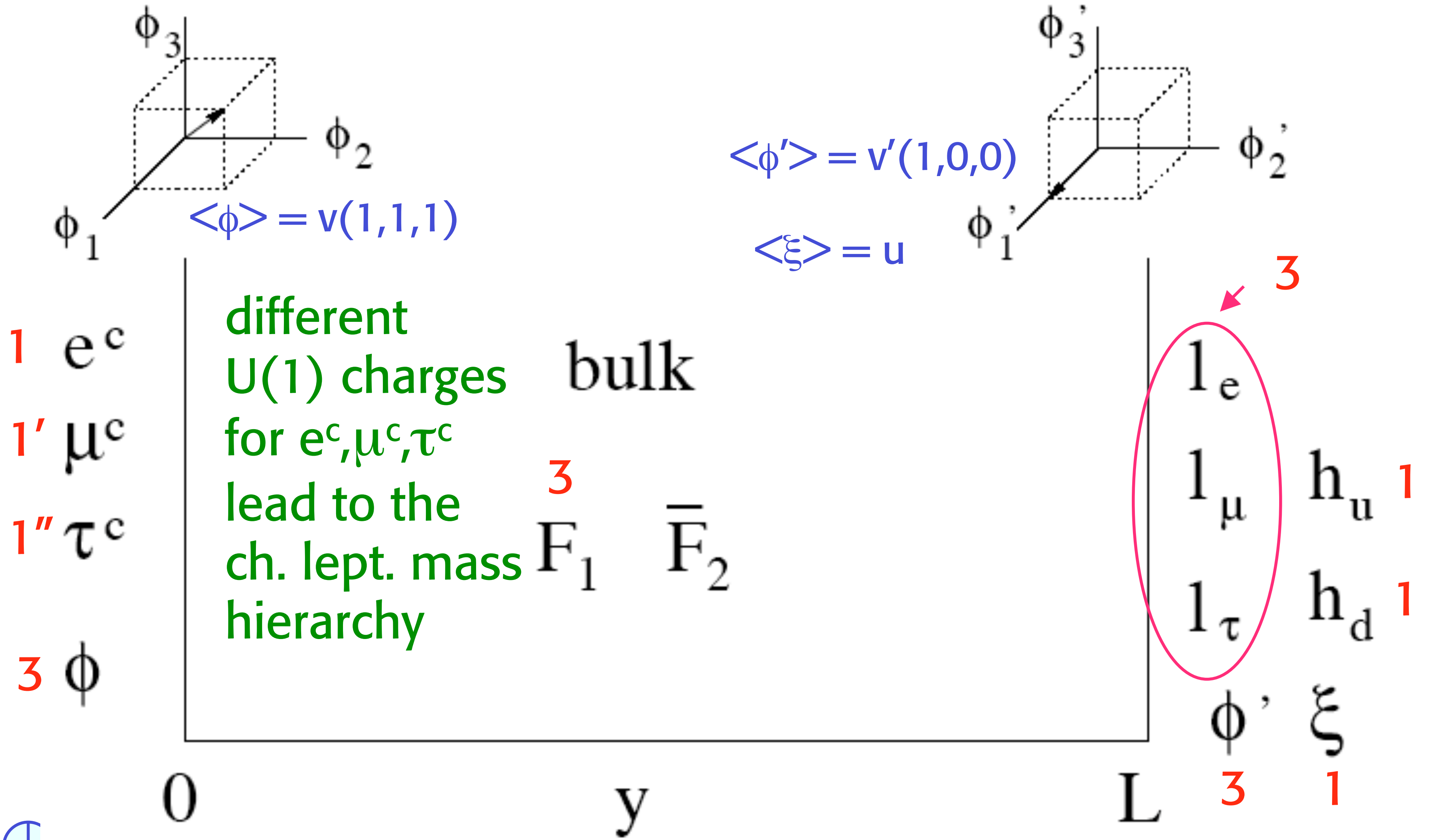
We have constructed two completely natural versions of the model:
- a version in 5 dimensions
- a SUSY version in 4-dim (with more fields)

We first briefly discuss the 5-dim version

GA, Feruglio hep-ph/0504165

The model has 1 compactified extra dim. and 2 branes

(crucial issue: guarantee and protect the vev alignment)



In lowest approximation the action is:

$$
\begin{aligned}
S &= \int d^4x dy \left\{ \left[iF_1 \sigma^\mu \partial_\mu \overline{F}_1 + iF_2 \sigma^\mu \partial_\mu \overline{F}_2 + \frac{1}{2}(F_2 \partial_y F_1 - \partial_y F_2 F_1 + h.c.) \right] \right. \\
&- M(F_1 F_2 + \overline{F}_1 \overline{F}_2) \\
&+ V_0(\varphi)\delta(y) + V_L(\varphi', \xi)\delta(y-L) \\
&+ \left[Y_e e^c(\varphi F_1) + Y_\mu \mu^c(\varphi F_1)'' + Y_\tau \tau^c(\varphi F_1)' + h.c. \right] \delta(y) \\
&+ \left. \left[\frac{x_a}{\Lambda^2}\xi(ll)h_u h_u + \frac{x_d}{\Lambda^2}(\varphi' ll)h_u h_u + Y_L(F_2 l)h_d + h.c. \right] \delta(y-L) \right\} + \ldots
\end{aligned}
$$

a Z-parity has also been imposed

$$
(f^c, l, F, \varphi, \varphi', \xi) \xrightarrow{Z} (-if^c, il, iF, \varphi, -\varphi', -\xi)
$$

After integrating out of the F fields one obtains the required effective 4-dim action

$$
\mathcal{L}_Y = y_e e^c(\varphi l) + y_\mu \mu^c(\varphi l)'' + y_\tau \tau^c(\varphi l)' + x_a \xi(ll) + x_d(\varphi' ll) + h.c. + \ldots
$$

In the flavour basis:

$$m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} a+2d/3 & -d/3 & -d/3 \\ -d/3 & 2d/3 & a-d/3 \\ -d/3 & a-d/3 & 2d/3 \end{pmatrix}$$

m_ν = U diag(a+d,a,-a+d)U^T (in units of v_u^2/Λ) and U=U_{HPS}

In terms of physical param.s (moderate normal hierarchy):

$$|m_1|^2 = \left[-r + \frac{1}{8\cos^2\Delta(1-2r)}\right]\Delta m^2_{atm} \sim (0.017\ \text{eV})^2$$

$$|m_2|^2 = \frac{1}{8\cos^2\Delta(1-2r)}\Delta m^2_{atm} \sim (0.017\ \text{eV})^2$$

$$|m_3|^2 = \left[1 - r + \frac{1}{8\cos^2\Delta(1-2r)}\right]\Delta m^2_{atm} \sim (0.053\ \text{eV})^2$$

- A moderate fine tuning is needed for r

A version with see-saw is also possible

ν_R is a triplet of A4: $\nu^c \sim 3$ No change for ch leptons

Dirac ↓ Majorana ↓

$$w_l = \ldots + y(\nu^c l) + x_A \xi(\nu^c \nu^c) + x_B(\varphi_T \nu^c \nu^c)$$

[Discrete parity Z: $\omega, \omega^2, \omega^2, \omega^2$ for l, ν^c, ϕ_T, ξ respectively]

$m_\nu^D \sim 1$

$$M_{RR} \sim \begin{bmatrix} A & 0 & 0 \\ 0 & A & D \\ 0 & D & A \end{bmatrix}$$

$m_\nu = m_\nu^{DT} M_{RR}^{-1} m_\nu^D \sim M_{RR}^{-1}$

The mass matrix appears just as the inverse of what was before, so that the mixing matrix is the same.

Eigenvalues are the inverse: one can produce inverse hierarchy with realistic θ_{12}, θ_{23} and very small θ_{13}

The model crucially depends on the precise vev alignment

$$\begin{aligned} \langle \varphi' \rangle &= (v', 0, 0) \\ \langle \varphi \rangle &= (v, v, v) \\ \langle \xi \rangle &= u \end{aligned}$$

The extra dimension with 2 branes allows the decoupling of the ϕ and ξ,ϕ' potentials.

A discrete symmetry is also essential: a separate continuous rotation symmetry on the 2 branes would make any disalignment illusory.

An alternative in 4 dimensions is a SUSY model with driving fields and a superpotential where all terms allowed by symmetry are present (with added fields ξ', ϕ_0, ϕ'_0, ξ_0).

In our models

- all terms allowed by symmetry are present
- all correct'ns are under control and can be made negligible

The 4-dim SUSY version (written in the T-diag basis)

In this basis the ch. leptons are diagonal!

$$w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + (x_a \xi + \tilde{x}_a \tilde{\xi})(ll) + x_b(\varphi_S ll) + h.c. + \ldots$$

One more singlet is needed for vacuum alignment

The superpotential (at leading order):

$$\begin{aligned} w_d &= M(\varphi_0^T \varphi_T) + g(\varphi_0^T \varphi_T \varphi_T) \\ &+ g_1(\varphi_0^S \varphi_S \varphi_S) + g_2 \tilde{\xi}(\varphi_0^S \varphi_S) + g_3 \xi_0(\varphi_S \varphi_S) + g_4 \xi_0 \xi^2 + g_5 \xi_0 \xi \tilde{\xi} + g_6 \xi_0 \tilde{\xi}^2 \end{aligned}$$

and the potential $V = \sum_i \left| \frac{\partial w}{\partial \phi_i} \right|^2 + m_i^2 |\phi_i|^2 + \ldots$

The assumed simmetries are summarised here

Field	l	e^c	μ^c	τ^c	$h_{u,d}$	φ_T	φ_S	ξ	$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A_4	3	1	$1'$	$1''$	1	3	3	1	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω	1	ω	ω
$U(1)_R$	1	1	1	1	0	0	0	0	0	2	2	2
$U(1)_F$		2q	q	1								

The driving field have zero vev. So the minimization is:

$$\frac{\partial w}{\partial \varphi_{01}^T} = M\varphi_{T1} + \frac{2g}{3}(\varphi_{T1}^2 - \varphi_{T2}\varphi_{T3}) = 0 \qquad \frac{\partial w}{\partial \varphi_{01}^S} = g_2\tilde{\xi}\varphi_{S1} + \frac{2g_1}{3}(\varphi_{S1}^2 - \varphi_{S2}\varphi_{S3}) = 0$$

$$\frac{\partial w}{\partial \varphi_{02}^T} = M\varphi_{T3} + \frac{2g}{3}(\varphi_{T2}^2 - \varphi_{T1}\varphi_{T3}) = 0 \qquad \frac{\partial w}{\partial \varphi_{02}^S} = g_2\tilde{\xi}\varphi_{S3} + \frac{2g_1}{3}(\varphi_{S2}^2 - \varphi_{S1}\varphi_{S3}) = 0$$

$$\frac{\partial w}{\partial \varphi_{03}^T} = M\varphi_{T2} + \frac{2g}{3}(\varphi_{T3}^2 - \varphi_{T1}\varphi_{T2}) = 0 \qquad \frac{\partial w}{\partial \varphi_{03}^S} = g_2\tilde{\xi}\varphi_{S2} + \frac{2g_1}{3}(\varphi_{S3}^2 - \varphi_{S1}\varphi_{S2}) = 0$$

$$\frac{\partial w}{\partial \xi_0} = g_4\xi^2 + g_5\xi\tilde{\xi} + g_6\tilde{\xi}^2 + g_3(\varphi_{S1}^2 + 2\varphi_{S2}\varphi_{S3}) = 0$$

Solution:

$$\varphi_T = (v_T, 0, 0) \quad , \qquad v_T = -\frac{3M}{2g}$$

$$\tilde{\xi} = 0$$

$$\xi = u$$

$$\varphi_S = (v_S, v_S, v_S) \quad , \qquad v_S^2 = -\frac{g_4}{3g_3}u^2$$

In the paper w at NLO is also studied

NLO corrections studied in detail

to m_l — 1st non trivial correction at $o(1/\Lambda^3)$

LO is $1/\Lambda$

to m_ν

$$\frac{x_c}{\Lambda^3}(\varphi_T\varphi_S)'(ll)''h_uh_u \qquad \frac{x_d}{\Lambda^3}(\varphi_T\varphi_S)''(ll)'h_uh_u \qquad \frac{x_e}{\Lambda^3}\xi(\varphi_T ll)h_uh_u$$

LO is $1/\Lambda^2$

to vevs

$$\begin{array}{lcl} \langle\varphi_T\rangle & \rightarrow & (v_T' + \delta v_T, \delta v_T, \delta v_T) \\ \langle\varphi_S\rangle & \rightarrow & (v_S + \delta v_1, v_S + \delta v_2, v_S + \delta v_3) \\ \langle\xi\rangle & \rightarrow & u \\ \langle\tilde{\xi}\rangle & \rightarrow & \delta u' \end{array}$$

LO is 1

δv_T, δv_S, δv_i, $\delta u' \sim o(1/\Lambda)$

All observables get a correction of order $1/\Lambda$

From exp (eg θ_{12}) must be less than 5%

$$0.0022 < \frac{v_S}{\Lambda} \approx \frac{v_T}{\Lambda} \approx \frac{u}{\Lambda} < 0.05$$

In particular $\theta_{13} < \sim 0.05$, $|tg^2\theta_{23} - 1| < \sim 0.05$

Extension to quarks

If we take all fermion doublets as 3 and all singlets as 1, 1′, 1″ (as for leptons): $Q_i \sim 3$, $u^c, d^c \sim 1$, $c^c, s^c \sim 1'$, $t^c, b^c \sim 1''$

Then u and d quark mass matrices are BOTH diagonalised by

$$U_u, U_d \sim \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}$$

As a result VCKM is unity: $V_{CKM} = U_u^+ U_d \sim 1$

So, in first approx. (broken by loops and higher dim operators), ν mixings are HPS and quark mixings ~identity

Corrections are far too small to reproduce quark mixings eg λ_C (for leptons, corrections cannot exceed $o(\lambda_C^2)$. But even those are essentially the same for u and d quarks)

Note: it not possible to embed this in a GUT:
with these assignments A4 does not commute with SU(5)

If l ~ 3 then all 5bar ~3, so that d^c_i ~ 3
if e^c, μ^c, τ^c ~ 1, 1', 1" then all 10_i ~ 1, 1', 1"

Realistic quark mass matrices are not easy to obtain from these assigments

For example, for u quarks at leading order:

$$m_u \sim 1.1 + 1'.1'' + 1''.1' \sim a\, u_1 u_1 + b\,(u_2 u_3 + u_3 u_2)$$

or

$$m_u \sim \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix}$$

Which implies $|m_c| = |m_t|$
and maximal U_{23}

Conclusion

From experiment: a good first approximation for quarks:

$$V_{CKM} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and for neutrinos

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Models based on A4 indeed lead to this pattern

All this is highly non trivial but no real illumination has followed!!

Main lessons from ν masses and mixings

- ν's are not all massless but their masses are very small
- probably masses are small because ν's are Majorana particles
- then masses are inv. prop. to the large scale M of L n. viol.
- $M \sim m_{\nu R}$ is empirically close to 10^{14}-10^{15} GeV ~ M_{GUT}
  -> ν masses fit well in the SUSY GUT picture
- decays of ν_R with CP & L violation can produce a B-L asymm.
  -> baryogenesis via leptogenesis
- detecting $0\nu\beta\beta$ would prove ν's are Majorana and L is viol.
- ν mixing angles are large except for θ_{13} that is small
- ν's are not a significant component of dark matter in Universe
- there is no contradiction between large ν mixings and small
  q mixings, even in GUT's