

Simplicity, Symmetry and Small Matrices

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Whatever may be one's opinion as to the simplicity of the laws of nature, there can be no question that the possessors of some such conviction have a real advantage in the race for physical discovery. Doubtless there are many simple connections still to be discovered, and he who has a strong conviction of the existence of these connections is much more likely to find them than he who is not at all sure they are there.

Percy W. Bridgman, 1927

These remarks by my predecessor at Harvard characterize my voyage of discovery through the world of elementary particles. While in High School, my classmates Gary Feinberg, Steve Weinberg and I made a precocious attempt to understand theoretical physics. In so doing, we recognized two complementary approaches: the one algebraic, the other analytic. This dichotomy is exemplified by the quantum theory of the hydrogen atom, which naturally separates into an 'angular' portion involving the directional coordinates of the electron, and a 'radial' portion involving the distance of the electron from the proton. The former problem involves angular momentum operators and is easily and elegantly dealt with algebraically, while the radial problem involves the solution to a differential equation.

Often one may classify theoretical physicists as either predominantly angular (like Heisenberg, with his matrix approach to quantum mechanics) or mostly radial (like Schroedinger, with his wave equation). Similarly, Richard Feynman was the more angular in his diagrammatic construction of quantum field theory, while Julian Schwinger was more radial or analytic. Early in my career, I opted for angles: for an approach to physics based on simplicity, symmetry and small matrices. Following this principle, I was fortunate to find several simple connections in the quest for the basic building blocks of matter and the rules by which they interact.

I was one of Schwinger's many students during the 1950s. My mentor pointed to two common features of weak and electromagnetic interactions that suggested their common origin. The equality of proton and electron charges, along with the descriptive success of the Puppi triangle, indicated that both interactions are universal. Moreover, both could be mediated by vector bosons. My Harvard thesis described a unified electroweak model based on the group $SO(3)$, in which 3×3 matrices would act on a triplet of leptons:

An $SO(3)$ Electroweak Scheme That Failed

$$\mathcal{J}_W \sim (\overline{\mu^+} \quad \bar{\nu} \quad \overline{e^-}) \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mu^+ \\ \nu \\ e^- \end{pmatrix}$$
$$\mathcal{J}_Q \sim (\overline{\mu^+} \quad \bar{\nu} \quad \overline{e^-}) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \mu^+ \\ \nu \\ e^- \end{pmatrix}$$

In this scheme, the leptons μ^+ , ν and e^- form a weak triplet. What is today called the muon neutrino is the electron's antineutrino. This $SO(3)$ model fails. However, in 1972 Georgi and I found a rather baroque electroweak model based on $SO(3)$... but by then, it was clearly not Nature's way.

My model incorporated Schwinger's view that muon and electron neutrinos must differ. "If we are to introduce a leptonic quantum number," he argued, "it should distinguish between electrons and muons of the same charge." Moreover, my late friend Gary Feinberg had pointed out that a weak-interaction theory with an intermediary boson demanded at least two neutrino states. But I digress. The fact is that my simple model failed. It predicted the wrong electron spectrum in muon decay, and it could not reconcile parity-violating weak interactions with parity-conserving electromagnetism.

I left Harvard in 1959 to postdoc in Copenhagen, at what would later be known as the Bohr Institute. While there, I found an appealing electroweak unification based on the group $SU(2) \times U(1)$, rather than $SO(3)$. The left-handed leptons (comprising two doublets) were to transform under simple 2×2 matrices, with the right-handed charged leptons being electroweak singlets:

The $SU(2) \times U(1)$ Scheme That Succeeded

Things were simpler than I had imagined. Nature uses even smaller matrices:

$$\mathcal{J}_W \sim (\overline{\nu_e} \quad \overline{e^-}) \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L + \dots$$
$$\mathcal{J}_Q \sim (\overline{\nu_e} \quad \overline{e^-}) \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} + \dots$$

The left-handed electron and its neutrino form a weak doublet. So also do the left-handed muon and its neutrino. So also does the third pair of leptons later to be found. The right-handed charged leptons are SU(2) singlets. But...

- How do the weak intermediaries acquire mass? Weinberg and Salam suggested a partial answer, but this remains a central question in particle physics that the LHC is designed to answer.
- How is the model to be extended to hadrons. and how are strangeness-violating neutral currents to be avoided? For this, we needed the quark hypothesis and the GIM mechanism.
- What about renormalizability? Gerard 'tHooft and Tini Veltman shared a Nobel Prize for solving this problem.

Of course, my model was not and could not be taken seriously until the later introduction of spontaneous symmetry breaking by Salam and Weinberg, and the quark model by Gell-Mann and Zweig, as extended to include charm.

Let's return to the 1950s and recall the puzzling state of particle physics. Pions and nucleons seemed adequate to describe nuclear processes, but how did strange particles fit in? Many investigators sought a higher symmetry that included isospin and strangeness, but could describe the curious pattern of baryons and mesons. Schwinger opted for 'Global Symmetry,' a scheme that implicitly assumed symmetry of pion interactions under SO(4) (along with a discrete operation). The Σ and Λ particles formed one 4-dimensional representation, nucleons and Ξ another. Tiomno chose the group SO(7), while Behrends and Sirlin favored the exceptional Lie group G₂. Meanwhile, Sakata proposed a model based on SU(3), with the nucleons and Λ forming a triplet. The remaining baryons, Σ and Ξ , formed an incomplete sextet. None of these schemes seemed to be correct.

Finally, in 1961, while I was a postdoc at CalTech, Gell-Mann (and, independently, my recently deceased friend Yuval Neeman), came up with the right idea. Like Sakata, they chose the symmetry group to be SU(3). Baryons and mesons, however, were both assigned to its 8-dimensional representation.

Higher Symmetries 1957–1961

Eight spin-1/2 baryons had been discovered. Along with nucleons, there were the hyperons: three Σ 's and the Λ^0 , along with two doubly-strange Ξ particles. Could these be described in terms of a more inclusive symmetry group that includes isospin and hypercharge? Many possible schemes were proposed:

<i>Lie Group</i>	<i>Baryon Rep</i>	<i>Proposer(s)</i>
$SO(7)$	8	J. Tiomno
$SO(4)$	$4 \oplus 4$	J. Schwinger
G_2	$7 \oplus 1$	R. Behrends & A. Sirlin
$SU(3)$	$3 \oplus 6$	S. Sakata
$SU(3)$	8	M. Gell-Mann & Y. Néeman

The Unitary Symmetry scheme won the competition, with spin-1/2 baryons and pseudoscalar mesons both assigned to octuplets of the Eight-Fold Way.

It was the right idea, but it would take several years to become generally accepted. Sidney Coleman and I, however, became immediate and ardent disciples of the Eightfold Way. We saw how to describe the theory with simple 3×3 matrices, rather than Murray's cumbersome 8×8 matrices. Using our trick, we deduced a remarkably successful mass formula involving baryon electromagnetic mass splittings (whose importance is limited today to being an exercise for beginning graduate students.)

Coleman-Glashow Ansatz

According to the eight-fold way, the 8 spin-1/2 baryons (as well as the soon-to-be 8 pseudoscalar mesons) formed supermultiplets. Their transformation behavior was expressed in terms of 8×8 matrices. But we found a simpler notation, with the baryons assembled into a much smaller 3×3 matrix:

$$\begin{pmatrix} -\sqrt{2/3} \Lambda & \Sigma^+ & p \\ \Sigma^- & \sqrt{1/6} \Lambda + \sqrt{1/2} \Sigma^0 & n \\ \Xi^- & -\Xi^0 & \sqrt{1/6} \Lambda - \sqrt{1/2} \Sigma^0 \end{pmatrix}$$

From which Sidney Coleman and I deduced a remarkably well satisfied relation among the electromagnetic mass splittings of the baryons:

$$M(\Sigma^-) - M(\Sigma^+) = M(\Xi^-) - M(\Xi^0) + M(n) - M(p)$$

$$8.08 \pm 0.08 \text{ MeV} = 7.77 \pm 0.24 \text{ MeV}$$

Sure enough, the various mesons and baryon states being discovered were fitted neatly into $SU(3)$ multiplets. The discovery in 1964 of the predicted Ω^- particle by the Samios group

convinced the community of the utility of the Eightfold Way, and soon afterward its success was attributed to the quark substructure of hadrons.

Meanwhile, Nicola Cabibbo had shown how to extend the weak current to describe hadrons, while preserving a modified form of universality. His weak current involved what became known as the Cabibbo angle.

**Cabibbo's Weak Hadronic Current
...And Its Neutral Counterpart**

$$\mathcal{J}_W \sim (u \quad d \quad s) \begin{pmatrix} 0 & \cos \theta & \sin \theta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$$\mathcal{J}_Q \sim (u \quad d \quad s) \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\cos^2 \theta & -\sin \theta \cos \theta \\ 0 & -\sin \theta \cos \theta & -\sin^2 \theta \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

In other words u and d_θ form a weak doublet, while s_θ remains a singlet (where d_θ and s_θ are linear combinations of d and s).

- The model lacks lepton-quark symmetry!
- The strangeness-*violating* neutral current leads to trouble, and would destroy any hope for an electroweak synthesis.

Cabibbo's model implied that the quarks relevant to the weak interactions are linear combinations of d and s . One linear combination, together with the up quark, forms a weak doublet. The other linear combination is excluded from the weak interactions.

On a return to Copenhagen in 1964, James Bjorken and I proposed (and named) the charmed quark. We saw that its introduction could restore lepton-quark symmetry to the theory. With the introduction of one more quark, the weak interactions would involve two doublets of leptons and two similar doublets of quarks. (A decade or so later, we would find that there are three doublets of each.)

Six years after the charm hypothesis, John Iliopoulos, Luciano Maiani and I realized that charm had much more than merely aesthetic appeal. Its introduction would solve the problem of strangeness-changing neutral currents, thus clearing the way for the electroweak theory.

The GIM Current & Its Neutral Counterpart

$$\mathcal{J}_W \sim (u \quad c \quad d \quad s) \begin{pmatrix} 0 & 0 & \cos \theta & \sin \theta \\ 0 & 0 & -\sin \theta & \cos \theta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u \\ c \\ d \\ s \end{pmatrix}$$

$$\mathcal{J}_0 \sim (u \quad c \quad d \quad s) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} u \\ c \\ d \\ s \end{pmatrix}$$

- Now the quarks form two weak doublets, just as do the leptons. What could be neater?
- With charm, the electroweak theory is easily extended to the hadron sector, without incurring horrid strangeness-changing neutral currents.

My next adventure involved 5×5 matrices. Howard Georgi and I proposed SU(5) as the unifying group of all elementary particle interactions — strong, weak and electro-magnetic. It was, I must say, a beautiful concept, but it posed one potential problem: it predicted proton decay. However, thanks to the work of Howard and his collaborators, the rate of proton decay was predicted to lie several orders of magnitude below its then-measured limit.

Grand Unified Theory

“We [propose] that SU(5) is the gauge group of the world— that all elementary-particle forces (strong, weak and electrodynamic) are different manifestations of the same fundamental interaction.” ... H. Georgi & S.L. Glashow (1973)

And so, we are led to 5×5 matrices, with the leptons and quarks of each family gathered into a vector and an antisymmetric tensor:

$$\begin{pmatrix} \bar{d}_r \\ \bar{d}_g \\ \bar{d}_b \\ \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} 0 & \bar{u}_b & -\bar{d}_g & d_r & u_r \\ \cdot & 0 & \bar{u}_r & d_g & u_g \\ \cdot & \cdot & 0 & d_b & u_b \\ \cdot & \cdot & \cdot & 0 & e^+ \\ \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix}_L$$

We know today that this is not quite the correct theory. But many investigators favour one or another of its super-symmetrized versions.

The story is too long to tell, but the upshot is that proton decay was looked for, but it was not observed at its predicted rate. Indeed, it has not been observed at all. Clearly, our

theory, as presented, is false. But I still believe that grand unification in some form, perhaps in a supersymmetrized version, is just too gorgeous a notion to abandon.

I conclude with a brief mention of ongoing work with my BU colleague Andrew Cohen. We conjecture that the space-time symmetry group of nature may be smaller than the Poincaré group. In particular, we assume it to be a certain subgroup of the Lorentz group, along with space-time translations. Our favorite choice is SIM(2), the largest proper subgroup of the Lorentz Group. We call our theory Very Special Relativity or VSR. But, you ask, how dare we abandon the Lorentz group? Hasn't the special theory of relativity been tested to an exquisite precision? Indeed it has, but all of the simple consequences of special relativity are preserved in VSR. Furthermore, VSR invariance implies Lorentz invariance in the limit of CP conservation, and CP is nearly an exact symmetry of Nature. Thus, the departures from exact Lorentz invariance are expected to be tiny.

And what sets the scale of the Lorentz-violating effects? Perhaps the neutrino masses, we suggest. As a trial hypothesis, we imagine that neutrino masses find their origin in VSR. My last slide explains what this all has to do with small matrices.

Very Special Relativity

We conclude with a return to 2×2 matrices, in the context of my ongoing work with Andrew Cohen. The Lorentz group — the basis to the special theory of relativity — is isomorphic to $SL(2C)$, the group of matrices of the form:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \text{where } a, b, c, d \text{ are complex} \\ \text{and } ac - bd = 1$$

The largest proper subgroup of the Lorentz group, SIM(2), is given by the matrices:

$$\begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \quad \text{where } a, b, d \text{ are complex} \\ \text{and } ad = 1$$

Can SIM(2) be the exact symmetry group of Nature, rather than the Lorentz group?

- Is neutrinoless double beta decay forbidden?
- Is the tritium endpoint spectrum deformed?
- Do electrons have transverse electric dipole moments?
- Do their magnetic moments undergo daily variation?

And so I have come to the end of my talk. My collaborators and I have used small matrices to design the electroweak model, to exploit the unitary symmetry scheme, to exhibit the need for charm and the GIM mechanism, to invent grand unification, and to search for Lorentz violation. They have served us well.